Meaning 4.1: Semantics 2 — Model-theoretic semantics

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Bringing our take-homes to lecture...

Assumption 1: Meaning is truth-conditional.

Intuition: To understand the meaning of a sentence we need to understand the conditions under which it is true.

Assumption 2: Meaning is compositional.

Intuition: To understand the meaning of a sentence we need to understand the meaning of its parts which can combine; additionally, we need to know how they combine.

Further subdividing

Research in semantics can be divided into the following:

(i) **Lexical semantics:** the meanings of the individual words/morphemes.

(ii) **Compositional semantics:** the rules for how the meanings fit together to form the meanings of sentences.

Model-theoretic semantics

We will study semantics from a model-theoretic perspective. Here are the main ideas:

(i) Words are represented by mathematical objects (typically, elements, ordered pairs or sets of elements/ordered pairs...) in our 'model world'

(ii) We model complex meanings by doing operations on the mathematical objects which correspond to words

(iii) Things we say are 'true' are those relationships which hold in our model

Model-theoretic semantics — intuition

Here is the logic:

(i) We use language to talk about things in the world; so, we create a model universe to represent these things. We represent things in the world by things in the model.

(ii) When we talk about things in the real world, we are often making reference to them directly or speaking about relations between them; these relations are modeled by operations on the things in the model (which represent things in the world).

(iii) We observe the behavior of things in the model and operations on those things; ideally, it should reflect how meaning works in language.

Analogy to arithmetic

Take the following:

$$2 \qquad + \qquad 8 \qquad = \qquad 10$$

Idea: We know the meaning of the elements $2, 8$, and we how to combine them, $+$, to make something, 10, with a new meaning — a meaning which uses information from the elements $2, 8$.

2, 8 introduce information (truth conditions in our case) and $+$ is an operation (compositionality in our case) on the pieces of information from 2, 8

Our case

We map linguistics units (typically words) onto a mathematical object.
The choice of mathematical object is a theoretical one; we choose ones which have properties we understand and ideally reflect the behavior of the linguistic unit.

When we map a word *exampleWord* on to the representing object in our model, we notate it by $[\![exampleWord]\!]$.

exampleWord is the linguistic unit; $[\![exampleWord]\!]$ is a mathematical object

Model-theoretic semantics with set theory

The type of mathematical framework we will work with is **set theory**.

The basic units are the following:

(i) **Elements**: atomic units

denote with lower-case letters $a, b, c, \ldots$

(ii) **Ordered pairs**: Pairs of elements where the order matters

denote a pair as $\langle a, b \rangle$

(iii) **Sets**: groups of elements or ordered pairs

denote with upper-case letters $A, B, C, \ldots$

Model-theoretic semantics with set theory

Finally, we have operations on and relationships between our basic units.

(i) **Membership**: If a is in a set A, we represent it with $a \in A$; if a set B is empty, we use the empty set symbol $\varnothing = B$

(ii) **Operations on and relationships between sets**:

A intersects B:	$A \cap B$	$a \in A$ **and** $a \in B$
A unions B:	$A \cup B$	$a \in A$ **or** $a \in B$
A is a subset of B:	$A \subseteq B$	if $a \in A$, then $a \in B$
A is a superset of B:	$A \supseteq B$	if $a \in B$, then $a \in A$

Tying it together

Lexical semantics: How do we map words on to set theory objects (elements, sets, ordered pairs)?

TRUTH CONDITIONS OF COMPONENTS
(TRUTH CONDITIONS)

Compositional semantics: How do we combine objects using set operations to build new objects with the right combined meaning?

TRUTH CONDITIONS OF COMBINED UNITS
(COMPOSITIONALITY)

Semantics of names — elements

The meaning of ‘Frege’ is the element (or also called individual) Frege (we can call it f) in our model. This follows more generally for referring expressions.

$[\![\text{Frege}]\!] = f =$

How about other parts of speech such as verbs, adjectives and nouns?

Semantics of adjectives, intransitive verbs and nouns — sets of individuals

Adjectives, intransitive verbs or nouns can pick out a **set of individuals** in our model we assume to have the properties of those adjectives, verbs or nouns

So, each of these words creates a partition of the universe of individuals into two sets, one with individuals that have that property and the other of individuals which don't

$[\![\text{tall}]\!] = \{x\text{: } x \text{ is tall}\}$

$[\![\text{philosopher}]\!] = \{x\text{: } x \text{ is a philosopher}\}$

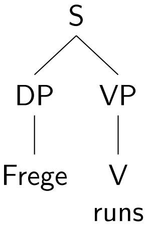
$[\![\text{run}]\!] = \{x\text{: } x \text{ runs}\}$

Now, the composition

The composition of meaning is dependent on the syntactic structure of the sentence.

For a sentence S with daughter nodes DP and VP from left to right, $[\![S]\!] =$ true iff $[\![DP]\!] \in [\![VP]\!]$

'Frege runs'



$[\![\text{Frege}]\!] = f$

$[\![\text{run}]\!] = \{x \colon x \text{ runs}\}$

The sentence is true iff f is a member of the set of runners.

Mathematically, true iff
$[\![\text{Frege}]\!] \in [\![\text{run}]\!]$
i.e. iff $f \in \{x \colon x \text{ runs}\}$

Closing observations

Note the following:

(i) The words *Frege* and *run* provided truth conditions
Truth condition 1: There is an individual, Frege. $[\![\text{Frege}]\!]$

Truth condition 2: There is a non-empty set of runners. $[\![\text{run}]\!]$

(ii) The syntactic structure of *Frege runs* and the model types of *Frege* (element/individual) and *run* (set of individuals) allowed for composition. $[\![\text{Frege}]\!] \in [\![\text{run}]\!]$

(iii) Meaning of the sentence is a complex set of truth conditions; sentence is true if and only if (iff) those conditions are satisfied in our model. $[\![\text{Frege runs}]\!] = \text{true iff } [\![\text{Frege}]\!] \in [\![\text{run}]\!]$

End of this video's material.